# A corpus-based toy model for DisCoCat

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Our semantics are free/minimal, in a certain sense explained later.

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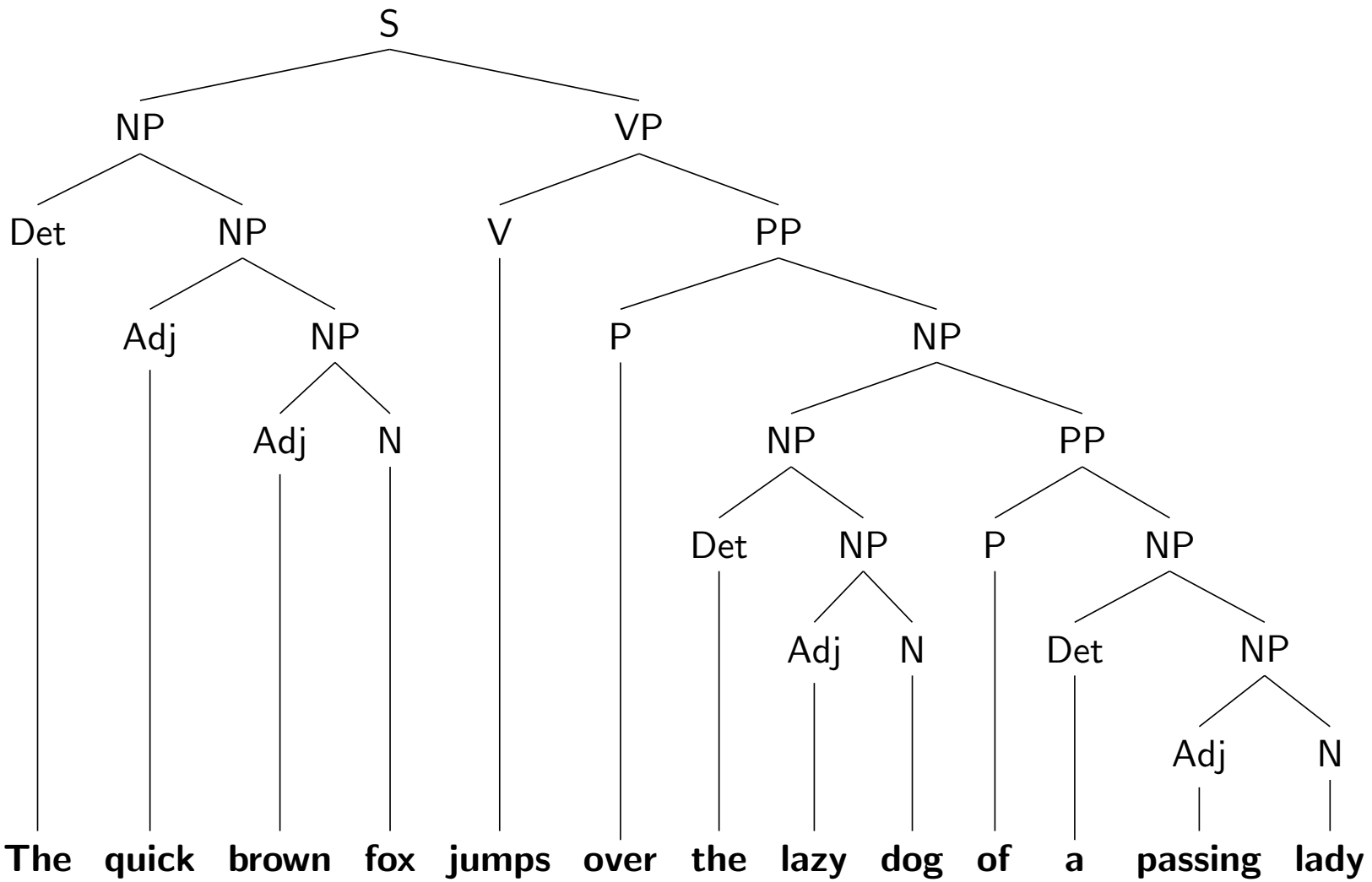
# Constituent Structure Trees



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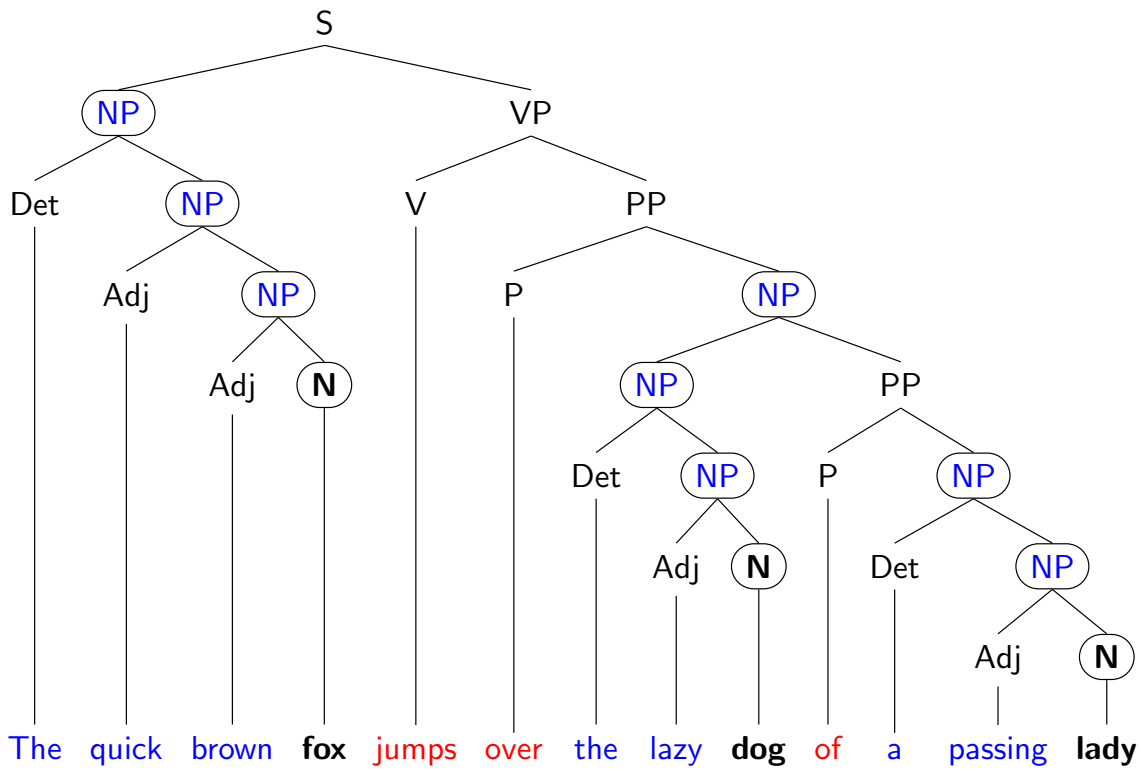
(iii) interaction fragments, of type $n^r \cdot n \cdot n^l$
- sentence fragments connecting noun phrases

# Objects and their Interactions




[1] jumps over [2,3]

[1]: the q. b. **fox** — quick brown **fox** — brown **fox** — **fox**

[2,3]: [2] of [3]

[2]: the lazy **dog** — lazy **dog** — **dog**

[3]: a passing **lady** — passing **lady** — **lady**

**Interaction words**

Modifying words

Interaction fragments

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[2] Including probability distributions and stochastic maps.

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- finite-dim convex cones[2], for $R = \mathbb{R}^+$
- finite multi-sets and "multi-relations", for $R = \mathbb{N}$

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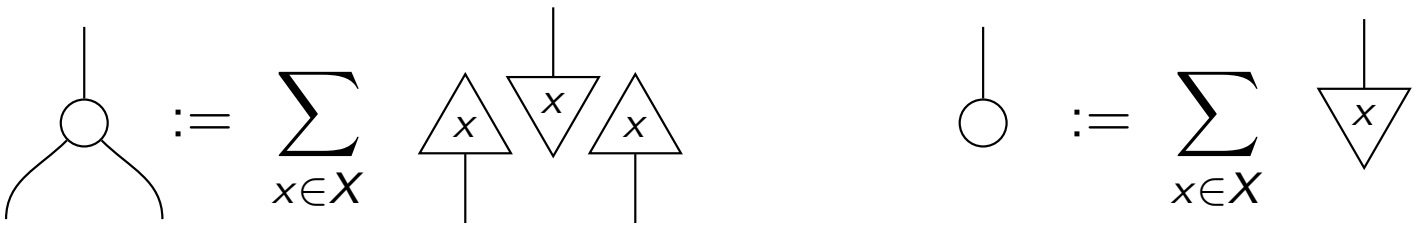
(iii) $R$-Mod has classical structures associated to canonical bases



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Semantics in any other semiring $R$ of interest can be recovered by covariance over the unique morphism $\mathbb{N} \to R$.

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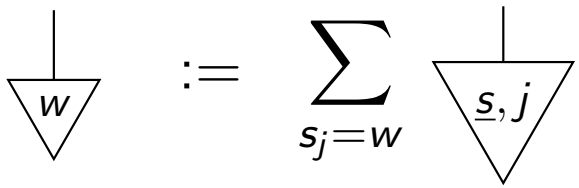
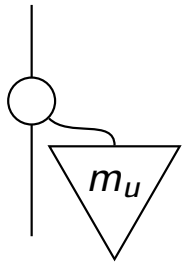
(iii) embed a word $w$ as the indicator function of all its instances

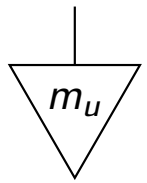


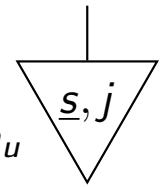
# The Compositional Part

Modifier words are mapped to projectors:

$$M_u := \quad \text{where} \quad m_u := \sum_{(\underline{s},j)\in m_u} \underline{s}, j$$

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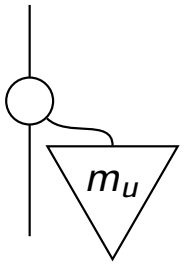
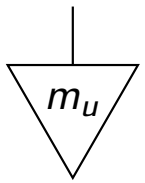
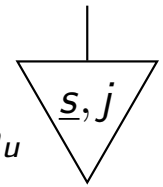
$M_u$ := 


where 


$:= \sum_{(\underline{s},j) \in m_u}$ 


We define $m_u$ to be the set of instances of object words which appear in objects modified by $u$. For example, from our sentence we'd have:

$$\{\text{fox}\} \subseteq m_{\text{quick}}$$
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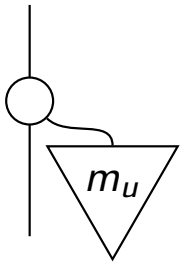
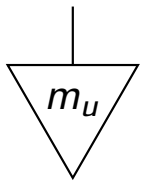
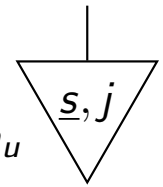
$M_u :=$ 


where 


$:= \sum_{(\underline{s},j)\in m_u}$ 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We automagically get some logic out of operator algebra[3].

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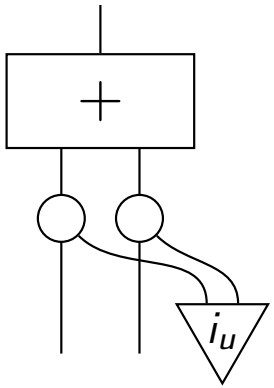
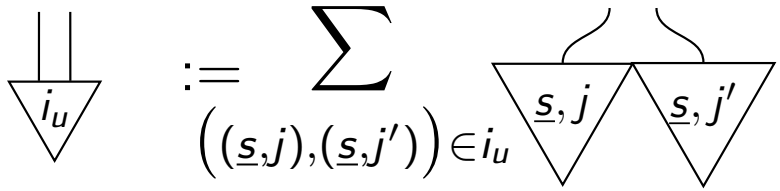
Interaction fragments are mapped to binary operations. First we construct the operation for single-word fragments:

$I_u :=$ 


where 

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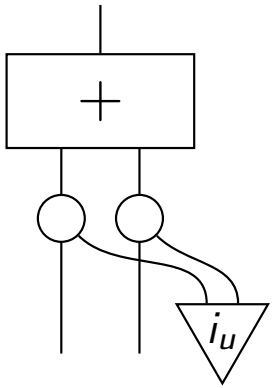
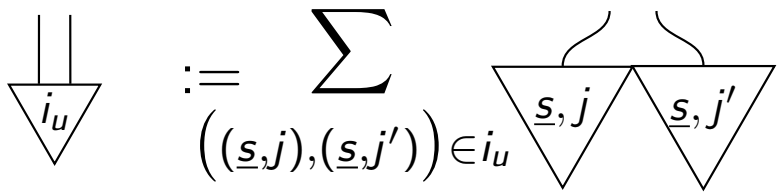
$I_u :=$ 


where 


We define $i_u$ to be the set of pairs of instances which appear in objects put into relation by $u$. For example, from our sentence we'd have:

$$\{(\text{fox,dog}), (\text{fox,lady})\} \subseteq i_{\text{jumps}} \tag{0.1}$$

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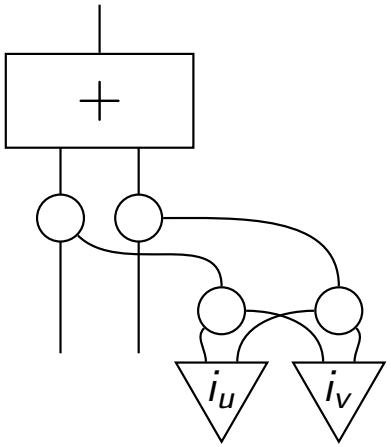
We use Frobenius algebras to treat multi-word fragments:

$$I_{(u,v)} = I_u \odot I_v :=$$



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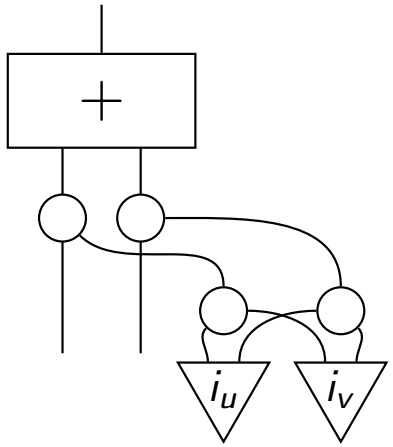


The linear operator $\boxed{+} : R^X \otimes R^X \to R^X$ is defined as follows:

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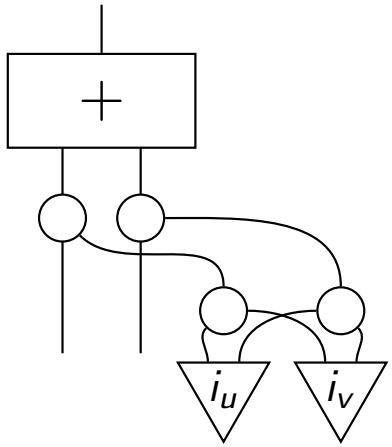


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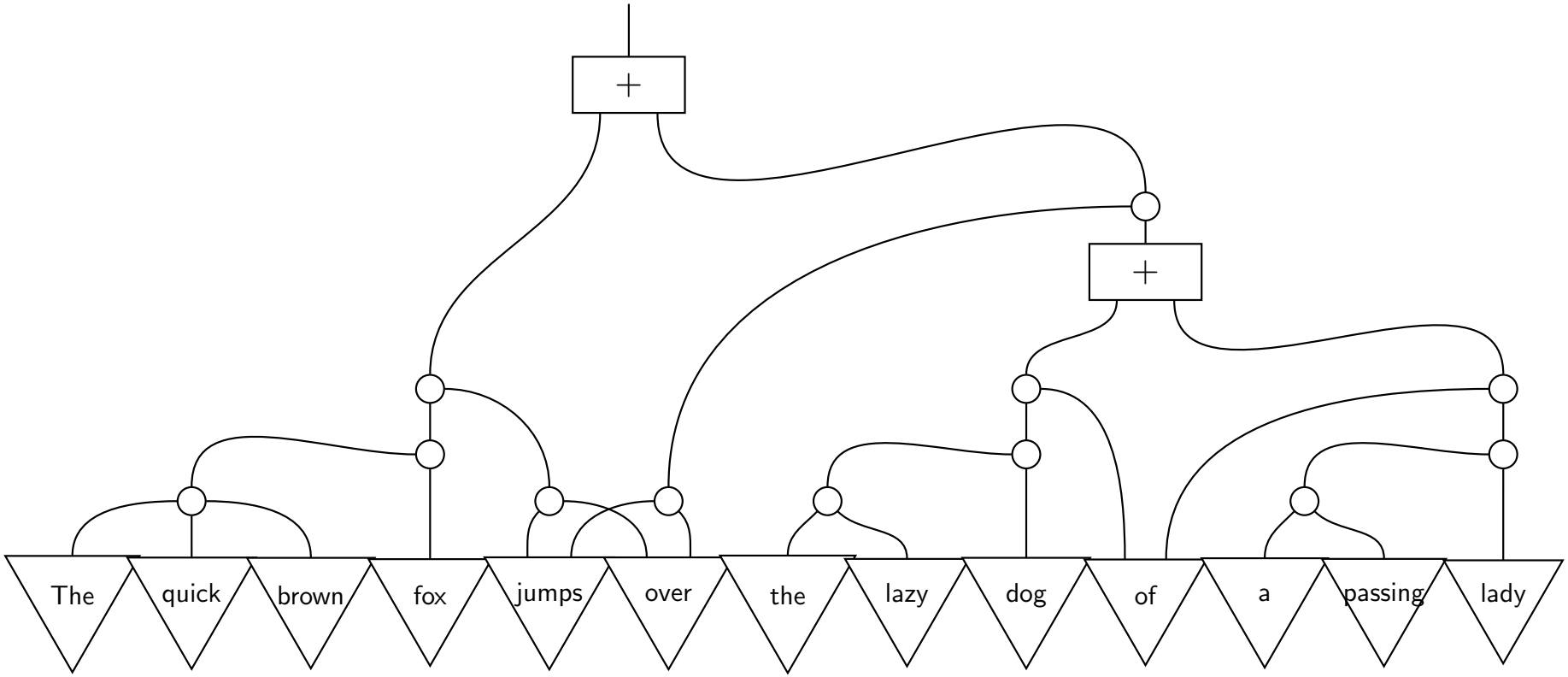
The interaction $I_{\underline{t}}$ sends a pair $w, w'$ of object words to the vector $|\psi\rangle$ spanning all object words appearing in objects which (i) contain instances of $w$ and $w'$, and (ii) are related by interaction fragment $\underline{t}$.

# The End Result

Here is the resulting[4] semantics for our sentence:




[4] After some applications of the spider theorem, to group modifier words together.

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(v) Enriched/higher order categories
- encode simplicial structure extracted from the corpus

# Thank You!

Thanks for Your Attention!

Any Questions?