

Quantifier Scope in Categorical Compositional Distributional Semantics

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In previous work with J. Hedges, we formalised a generalised quantifiers theory of natural language in categorical compositional distributional semantics with the help of bialgebras. In this paper, we show how quantifier scope ambiguity can be represented in that setting and how this representation can be generalised to branching quantifiers.

1 Introduction

Categorical Compositional Distributional semantics (CCDS) adds compositionality to distributional semantics via a functorial passage from the syntax to the semantics of natural language [4]. Both the syntax and the semantics are represented by compact closed categories. The claim is that regardless of how complex the structure of a sentence can be and what bizarre forms the words therein can take, as long as the sentence is represented in the syntax, CCDS will prescribe a vector to represent its semantics. In practice, however, one needs more than a syntax and a functorial passage. Semantic postulates for meanings of words are a requirement. As long as these are of good quality, CCDS will provide a good theoretical glue. Various distributional semantics can be trusted in their ability to produce vectors for atomic words; functional words, on the other hand, need a special treatments.

In our path to provide special treatments for functional words of natural language, in [10] we dealt with negation, in [8, 11] we showed how meanings of verbs and relative pronouns can be represented using Frobenius algebras. Recently, we used bialgebras to represent meanings of quantifiers [6]. Therein, we also developed a relational instantiation for the model and proved that it is sound and complete with regard to the generalised quantifier theory of natural language by Barwise and Cooper [2]. A natural question that arises in every quantifier theory is that of scope. The scope and branching of generalised quantifiers have been discussed in a wealth of literature, e.g. see the collection of papers by Gärdenfors [5], the paper by Westerstahl in that collection [12], and of course the original work of Barwise himself [1]. In this short account, we show how one may deal with scope and branching in our bialgebraic CCDS model of quantifiers.

2 Compact Closed Categories, Bialgebras, Examples, Diagrams

Recall that a non-symmetric compact closed category, \mathcal{C} , has objects A, B ; morphisms $f: A \rightarrow B$; and a monoidal tensor $A \otimes B$ that has a unit I , that is we have $A \otimes I \cong I \otimes A \cong A$. Furthermore, for each object A there are two objects A^r and A^l and the following morphisms:

$$A \otimes A^r \xrightarrow{\varepsilon^r} I \xrightarrow{\eta^r} A^r \otimes A \quad A^l \otimes A \xrightarrow{\varepsilon^l} I \xrightarrow{\eta^l} A \otimes A^l$$

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These morphisms satisfy four equalities, known by ‘yanking’, which we will not give here and the reader can check them here [4].

A bialgebra in a symmetric monoidal category $(\mathcal{C}, \otimes, I, \sigma)$ is a tuple $(X, \delta, \iota, \mu, \zeta)$ where, for X an object of \mathcal{C} , the triple (X, δ, ι) is an internal comonoid and (X, μ, ζ) is an internal monoid. This means that we have the following morphisms in \mathcal{C} :

$$\delta: X \rightarrow X \otimes X, \quad \iota: X \rightarrow I; \quad \mu: X \otimes X \rightarrow X, \quad \zeta: I \rightarrow X$$

where δ and ι are coassociative and counital and μ and ζ are associative and unital. δ and μ satisfy the following four equations [9]:

$$\iota \circ \mu = \iota \otimes \iota \tag{Q1}$$

$$\delta \circ \zeta = \zeta \otimes \zeta \tag{Q2}$$

$$\delta \circ \mu = (\mu \otimes \mu) \circ (\text{id}_X \otimes \sigma_{X,X} \otimes \text{id}_X) \circ (\delta \otimes \delta) \tag{Q3}$$

$$\iota \circ \zeta = \text{id}_I \tag{Q4}$$

Among examples of compact closed categories are category of sets and relations Rel and category of finite dimensional vector spaces and linear maps FdVect . Both Rel and FdVect are symmetric. Hence in these categories for any two objects A, B we have $A \otimes B \cong B \otimes A$. As a result, for any object A we obtain that $A^r = A^l = A^*$, for A^* a dual object. In Rel , for any set S , we have that $S^* \cong S$. In FdVect , this holds for vector spaces that have a fixed basis. In Rel , given a set S with elements $s_i, s_j \in S$, the epsilon and eta maps are given as follows:

$$\begin{aligned} \varepsilon = \varepsilon^l = \varepsilon^r: S \times S \rightarrow I, & \quad (s_i, s_j)\varepsilon \star \iff s_i = s_j \\ \eta = \eta^l = \eta^r: I \rightarrow S \times S, & \quad \star \eta(s_i, s_j) \iff s_i = s_j \end{aligned}$$

Here, \times is the tensor of the category with the singleton set $I = \{\star\}$ as its unit. In FdVect , given a basis $\{r_i\}_i$ for a vector space V , the epsilon and eta maps are as follows:

$$\begin{aligned} \varepsilon = \varepsilon^l = \varepsilon^r: V \otimes V \rightarrow \mathbb{R}, & \quad \varepsilon \left(\sum_{ij} c_{ij} (\psi_i \otimes \phi_j) \right) = \sum_{ij} c_{ij} \langle \psi_i | \phi_j \rangle \\ \eta = \eta^l = \eta^r: \mathbb{R} \rightarrow V \otimes V, & \quad \eta(1) = \sum_i (|r_i\rangle \otimes |r_i\rangle) \end{aligned}$$

Here \otimes is the tensor product of two vector spaces with \mathbb{R} as its unit.

As it will become apparent in the next section, generalised quantifiers are maps of the form $\mathcal{P}(U) \rightarrow \mathcal{P}\mathcal{P}(U)$, for U a universe of reference. Thus, the bialgebra structures that we are interested in are on powerset objects. In Rel , these are over objects of the form $\mathcal{P}(U)$. In FdVect , they are over vector spaces spanned by such objects, that is $V_{\mathcal{P}(U)}$. In Rel , for $A, B, C \subseteq U$, they are defined as follows:

$$\begin{aligned} \delta: \mathcal{P}(U) \rightarrow \mathcal{P}(U) \times \mathcal{P}(U) & \quad A\delta(B, C) \iff A = B = C \\ \iota: \mathcal{P}(U) \rightarrow \{\star\}, & \quad A\iota \star \iff \text{(always true)} \\ \mu: \mathcal{P}(U) \times \mathcal{P}(U) \rightarrow \mathcal{P}(U), & \quad (A, B)\mu C \iff A \cap B = C \\ \zeta: \{\star\} \rightarrow \mathcal{P}(U), & \quad \star \zeta A \iff A = U \end{aligned}$$

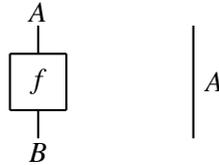
The coalgebraic δ map copies its input A into its two outputs B and C , so we have $A = B = C$. Its unit ι relates any subset A of U to the single element in $\{\star\}$. The algebraic μ map, takes two subsets A and B

and relates them to their intersection $A \cap B$. Its unit ζ relates the \star to the universe of reference. The above definitions become as follows in FdVect :

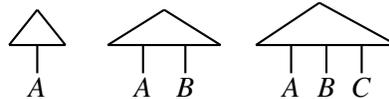
$$\begin{aligned} \delta: V_{\mathcal{P}(U)} &\rightarrow V_{\mathcal{P}(U)} \otimes V_{\mathcal{P}(U)} & \delta|A\rangle &= |A\rangle \otimes |A\rangle \\ \iota: V_{\mathcal{P}(U)} &\rightarrow \mathbb{R}, & \iota|A\rangle &= 1 \\ \mu: V_{\mathcal{P}(U)} \times V_{\mathcal{P}(U)} &\rightarrow V_{\mathcal{P}(U)}, & \mu(|A\rangle \otimes |B\rangle) &= |A \cap B\rangle \\ \zeta: \mathbb{R} &\rightarrow V_{\mathcal{P}(U)}, & \zeta &= |U\rangle \end{aligned}$$

where they have the same functionalities. δ copies its input basis vector and ι relates it to $1 \in \mathbb{R}$. μ sends its two input basis vectors to the basis vector obtained by taking the intersection of them. Recall that the basis vectors are subsets of U , hence their intersection is set intersection. In previous work [6], we showed how these definitions satisfy the bialgebra conditions (Q1) to (Q4).

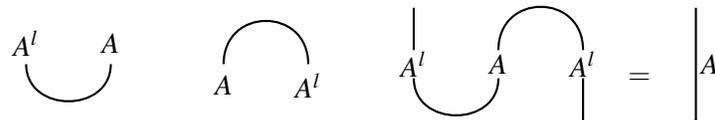
Diagrammatically, a morphism $f: A \rightarrow B$ and an identity on an object A of a compact closed category are depicted as follows:



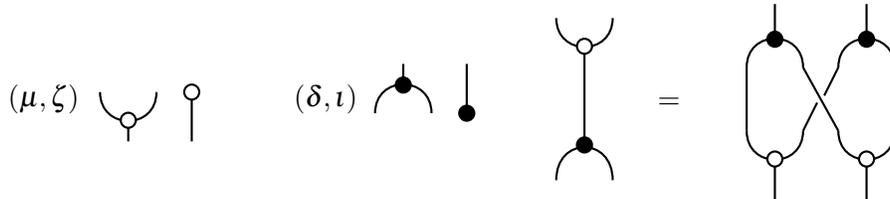
In concrete categories, morphisms of the form $I \rightarrow A$ represent elements of A . These are depicted as follows for elements of $A, A \otimes B$, and $A \otimes B \otimes C$; elements of other tensor objects are depicted similarly:



The ε and η maps are depicted by cups and caps, and yanking by straightening of curves. We have one of the following tuple of diagrams for each ε and η :



The diagrams for the bialgebraic monoid and comonoid morphisms and their interaction (the bialgebra law Q3) are as follows:



3 Generalised Quantifiers in Natural Language

A generalised quantifier q on a universe U is the image of a function of the following form:

$$q: \mathcal{P}(U) \rightarrow \mathcal{P}\mathcal{P}(U)$$

We sometimes abuse the terminology and call the function itself a quantifier. The q function can equivalently be represented by a relation over $\mathcal{P}(U)$, that is by a subset of $\mathcal{P}(U) \times \mathcal{P}(U)$. We use a category theoretical notation and denote this relation by a barred line as follows: $\mathcal{P}(U) \bar{\rightarrow} \mathcal{P}(U)$.

For A and X subsets of U , examples of this relation for first order quantifiers are as follows:

$$A \xrightarrow{\text{some}} X \iff X \cap A \neq \emptyset \qquad A \xrightarrow{\text{all}} X \iff A \subseteq X$$

Quantifiers that go beyond first order, for example ‘few, most, several, many’ are also definable. For α such a quantifier, we have:

$$A \xrightarrow{\alpha} X \iff |A \cap X| = \alpha \text{ elements of } A$$

where the definition of ‘ α elements of a set’ depends on one’s underlying model. The following property is of importance when using such quantifiers in natural language. A quantifier q is said to be *conservative* whenever $A \xrightarrow{q} X$ iff $A \xrightarrow{q} X \cap A$. If a quantifier q is conservative, it is said that q lives on A .

Syntactically, quantified phrases of natural language are generated via the following context free rules, where NP is a noun phrase, N is a common noun, and D is a determiner:

$$\text{NP} \rightarrow \text{D N} \qquad \text{N} \rightarrow \text{cat, dog, men, } \dots \qquad \text{D} \rightarrow \text{a, some, all, no, most, few, } \dots$$

In [2], Barwise and Cooper take the semantics of natural language to be a pair $(U, \llbracket \cdot \rrbracket)$, where U is a universe of reference and $\llbracket \cdot \rrbracket$ is a map from the vocabulary of the language to subsets or sets of subsets of U or products thereof. This assignment depends on the types of the words. The semantics of a verb phrase vp is a unary relation over U , that is $\llbracket vp \rrbracket \subseteq U$, that of a verb is a binary relation, that is $\llbracket v \rrbracket \subseteq U \times U$. The semantics of a determiner d is a map of the form $\llbracket d \rrbracket: \mathcal{P}(U) \rightarrow \mathcal{P}\mathcal{P}(U)$. Given $\llbracket n \rrbracket$ as the semantics of a noun n , that is $\llbracket n \rrbracket \subseteq U$, the semantics of a quantified noun $\llbracket d n \rrbracket$ is defined to be $\llbracket d \rrbracket(\llbracket n \rrbracket)$ and is a set of subsets, that is $\llbracket d n \rrbracket \subseteq \mathcal{P}\mathcal{P}(U)$. Here, d is a conservative generalised quantifier. To be conservative for d means that the following holds:

$$\llbracket n \rrbracket \xrightarrow{d} X \iff \llbracket n \rrbracket \xrightarrow{d} X \cap \llbracket n \rrbracket$$

The semantics of phrases and sentences are defined by induction over their generation structure. The semantics of a sentence s is referred to by ‘true’ whenever $\llbracket s \rrbracket \neq \emptyset$.

4 Bialgebraic Treatment of Generalised Quantifiers

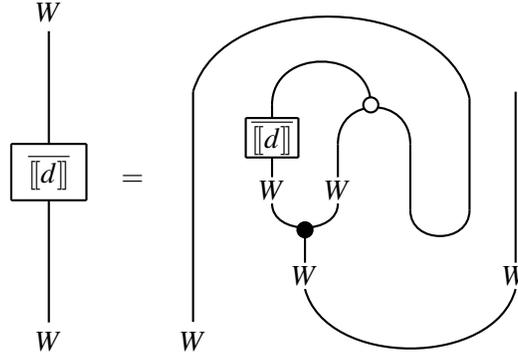
In previous work [6], we presented a compositional way of representing quantified phrases and sentences of natural language as morphisms of a self adjoint compact closed category \mathcal{C} that has two designated objects W and S , where W has a bialgebra structure over it. In order to make the syntax-semantics interface work, we started with a context free grammar, turned it into a pregroup grammar (via a known translation [3]), and defined a strongly monoidal functor $\llbracket \cdot \rrbracket$ between this pregroup grammar and \mathcal{C} as described above. We used the tuple $(\mathcal{C}, W, S, \llbracket \cdot \rrbracket)$ to represent this interface. In the current paper, we shall skip the syntactic details and only review the semantics.

A determiner d is interpreted using a morphism of the form $\llbracket d \rrbracket: W \rightarrow W$. The meaning of a determiner-noun phrase ‘ $d n$ ’, becomes $\llbracket d n \rrbracket = \llbracket d \rrbracket \circ \llbracket n \rrbracket$. In order to express conservativity of quantifiers, we need an intersection operator, so we use the bialgebraic internal monoid map μ . One needs to perform another action, other than intersection, on the input of the d map. We first need to apply the $\llbracket d \rrbracket$

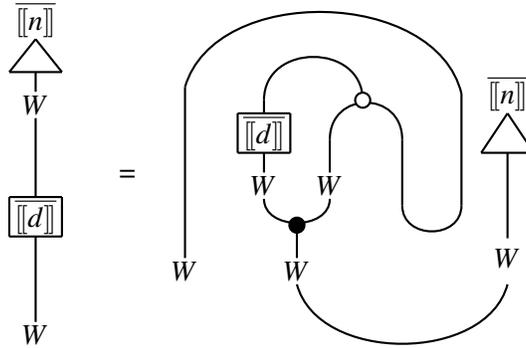
morphism to this input, and only then take its intersection with the result. So we need to first copy the input. For this, we use the bialgebraic comonoid map δ . With these considerations in mind, we defined a conservative quantifier to be the following morphism:

$$\overline{[d]} = (1_W \otimes \varepsilon_W) \circ (1_W \otimes \mu_W \otimes \varepsilon_W \otimes 1_W) \circ (1_W \otimes \overline{[d]} \otimes \delta_W \otimes 1_{W \otimes W}) \circ (1_W \otimes \eta_W \otimes 1_{W \otimes W}) \circ (\eta_W \otimes 1_W)$$

which is depicted as follows:



Taking the above into account, we obtain the following for the meaning of ‘d n’:



Using the dual of the bialgebra defined previously, the above can further simplify. or reasons of space, we do not give this simplification here and refer the reader to [6].

So far, we have worked with an abstract compact closed categorical setting and within tuples of the form $(\mathcal{C}, W, S, \overline{[]})$, as defined previously. This abstract setting can instantiate to provide concrete models. For instance, a *relational instantiation* of the abstract setting can be the tuple $(\text{Rel}, \mathcal{P}(\mathcal{U}), \{\star\}, \overline{[]})$ with $\mathcal{P}(\mathcal{U})$ and the singleton set $\{\star\}$ as its two designated objects and where $\mathcal{P}(\mathcal{U})$ has a bialgebra over it. This is the relational instantiation we used in previous work to prove an equivalence between the truth theoretic version of our semantics and that of Barwise and Cooper. Herein, we first defined meaning of a sentence s to be true iff $\star \overline{[s]} \star$. We then proved the following equivalence:

$$\star \overline{[s]} \star \quad \text{iff} \quad \overline{[s]} \neq \emptyset \quad (1)$$

That is, the meaning of a quantified sentence is true in the relational instantiation of our abstract categorical setting iff it is true in the generalised quantifier theory. Since sets and relations embed into vector spaces and linear maps (sets as vector spaces spanned by their elements and relations as linear maps corresponding to their tables), one can immediately obtain the following embedding of instantiations:

$$(\text{Rel}, \mathcal{P}(\mathcal{U}), \{\star\}, \overline{[]}) \quad \rightsquigarrow \quad (\text{FdVect}, \mathcal{V}_{\mathcal{P}(\mathcal{U})}, \mathcal{V}_{\{\star\}}, \overline{[]})$$

where FdVect here is the category of vector spaces equipped with fixed orthogonal bases indexed by finite sets from the universe U . In FdVect , the vector version of equivalence (1) holds, for details please see [6]. Since $V_{\{\ast\}} \cong \mathbb{R}$, meanings of sentences in this model become real numbers, interpretable as degrees of truth. In order to have vectors as meanings of sentences, we instantiated the model to tuples of the form $(\text{FdVect}, V_{\mathcal{P}(\Sigma)}, Z, \llbracket \cdot \rrbracket)$, where Z is a vector space wherein interpretations of sentences live. Again, for details please see [6].

5 Scope

For the sake of explaining the question of scope, consider a natural language sentence $d_1 n_1 v d_2 n_2$ with two determiners d_1 and d_2 , a subject n_1 , an object n_2 , and a verb v . Suppose the semantics of this sentence is represented in a logical form, where the words and their semantics are denoted by the same letter. So subject 1 and its semantics are both denoted by n_1 and similarly for the rest of the words. One faces two possibilities in the semantics regarding the scopes of the quantifications, as follows:

$$(1) d_1 x(n_1(x), q_2 y(n_2(y), v(x, y))) \quad (2) d_2 y(n_2(y), q_1 x(n_1(x), v(x, y)))$$

In (1), d_1 has a wide scope and d_2 a narrow scope, whereas in (2) d_2 has a wide scope and d_1 a narrow scope. As an example, consider the sentence ‘all men admire some cars’, which can be interpreted by either of the following two formulae:

$$(1) \forall x(\text{man}(x), \exists y(\text{car}(y), \text{admire}(x, y))) \quad (2) \exists y(\text{car}(y), \forall x(\text{man}(x), \text{admire}(x, y)))$$

Generalised quantifiers cannot in general be represented in first order logic and the above are not meant to be formulae of first order logic. To see this, note that we have used commas and not the usual conjunction/implication connectives inside the brackets.

Depending on the choice of quantifiers, the above two options might or might not provide the same semantics for the sentence. For instance, they will amount to the same meaning for the sentence “some men admire some cars”, whereas for the sentence “all men admire some cars”, or “all men admire two cars”, these interpretations result in two different meanings. For option (2) to be true, all men have to admire the same car in the sentence “all men admire some cars” and they have to admire the same two cars in the sentence “all men admire two cars”. Due to the presence of such ambiguities in natural language sentences, it is desirable that a semantic system can represent both of these interpretations. The question we address in what follows is whether our bialgebraic treatment of generalised quantifiers reflects this ambiguity and if so how.

When dealing with the question of scope, the form of the predicates, i.e. whether they are sets or vectors, relations or linear maps, is less important than the general pattern of the sentences containing them. This pattern is the same in any instantiation of the abstract categorical semantics. Having this mind, we start our treatment by working within the relational instantiation and at the end provide an abstract categorical solution using the diagrammatic representation.

In the relational instantiation, the above two readings are obtained by computing the results of the following two interpretations:

$$(1) \overline{\llbracket d_1 n_1 \rrbracket} (\overline{\llbracket v d_2 n_2 \rrbracket}) \quad (2) \overline{\llbracket d_2 n_2 \rrbracket} (\overline{\llbracket d_1 n_1 v \rrbracket})$$

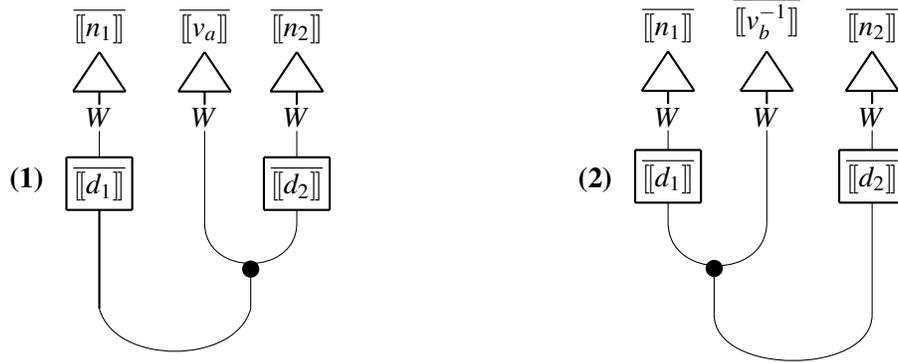
Unfolding the above provides us with the following:

$$(1) \llbracket n_1 \rrbracket \xrightarrow{\llbracket d_1 \rrbracket} \{a \in \llbracket n_1 \rrbracket \mid \llbracket d_2 n_2 v a \rrbracket\}, \text{ where for } \llbracket d_2 n_2 v a \rrbracket \text{ we have } \llbracket n_2 \rrbracket \xrightarrow{\llbracket d_2 \rrbracket} \{b \in \llbracket n_2 \rrbracket \mid a \xrightarrow{\llbracket v \rrbracket} b\}$$

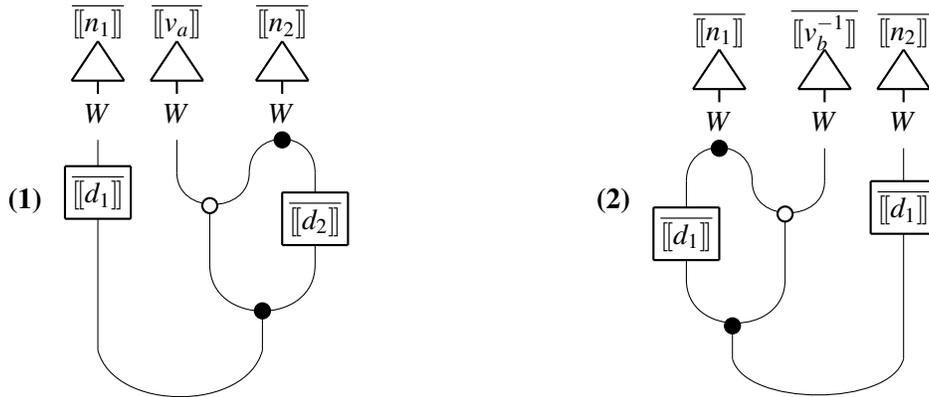
$$(2) \llbracket n_2 \rrbracket \xrightarrow{\llbracket d_2 \rrbracket} \{b \in \llbracket n_2 \rrbracket \mid \llbracket d_1 n_1 v b^{-1} \rrbracket\}, \text{ where for } \llbracket d_1 n_1 v b^{-1} \rrbracket \text{ we have } \llbracket n_1 \rrbracket \xrightarrow{\llbracket d_1 \rrbracket} \{a \in \llbracket n_1 \rrbracket \mid b \xrightarrow{\llbracket v \rrbracket^{-1}} a\}$$

In option (1), we first compute the n_2 's (i.e. $b \in \llbracket n_2 \rrbracket$), that are in the v relationship with an n_1 and check which one of these are in relationship d_2 with n_2 , that is, for example when this set is in $d_2(n_2)$. From these, we pick the elements whose n_1 's (i.e. $a \in \llbracket n_1 \rrbracket$) are in relationship d_1 with n_1 . In option (2), we do the same but in the opposite order: first pick n_1 's that are in the v relationship with an n_2 and check which one of them are related to $d_1(n_1)$, from these, we pick the ones whose n_2 is related to $d_2(n_2)$.

In the relational instantiation of the categorical setting, we worked with powersets and had our interpreted objects be of the same type: that is of type $\mathcal{P}(U)$. Hence, checking if a set is an element of another set and returning it as the result if this is the case, becomes equivalent to applying the bialgebraic comonoid operation δ of type $\mathcal{P}(U) \rightarrow \mathcal{P}(U) \times \mathcal{P}(U)$. In the abstract compact closed semantics, this is the comonoid map on the designated object W , that is $\delta: W \rightarrow W \times W$. Taking these into account, we obtain the following two diagrammatic interpretations for the above two readings:



In the relational instantiation, we defined $\llbracket z_x \rrbracket$, for z either v or v^{-1} and $x \in X$, to be the set $\{y \in Y \mid x \xrightarrow{\llbracket z \rrbracket} y\}$. The abstract compact closed form of this set is the morphism $I \xrightarrow{\llbracket v_x \rrbracket} Y$, derivable from the morphism $I \xrightarrow{\llbracket v \rrbracket} X \otimes Y$. By conservativity, computing the truth of $\llbracket d \ n \ v_x \rrbracket$ is the same as checking whether $\llbracket n \rrbracket \cap \llbracket v_x \rrbracket \in \llbracket d \rrbracket(\llbracket n \rrbracket)$. In the relational instantiation, we used the bialgebraic monoid map $\mu: \mathcal{P}(U) \times \mathcal{P}(U) \rightarrow \mathcal{P}(U)$ to model intersection. In the abstract categorical setting, this becomes the monoid map on W , and the above diagrams are unfolded as follows:



Here, since we are now doing two operations on the $\llbracket n_1 \rrbracket$ in reading (1) and on the $\llbracket n_2 \rrbracket$ in reading (2), we need to apply a μ to $\llbracket n_1 \rrbracket$ in reading (1) and to $\llbracket n_2 \rrbracket$ in reading (2), right at the beginning.

6 Branching

Branching of quantifiers happens when there is a partial ordering on them. Henkin's prefix quantifiers are a form of branching. In the sentence $d_1 n_1 v d_2 n_2$, the partial order between two quantifiers is depicted as follows:

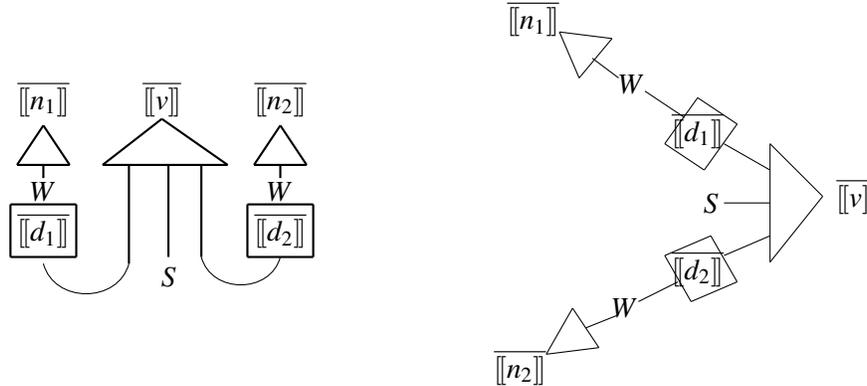
$$\begin{array}{c} d_1 x n_1(x) \\ \quad \quad \quad \searrow \\ \quad \quad \quad v(x,y) \\ \quad \quad \quad \nearrow \\ d_2 y n_2(y) \end{array}$$

When d_1 and d_2 have a linear ordering between them, the above unfolds to the two readings discussed in the previous section. The scope options seem to have resulted from having different linear orderings on quantifiers. This was recognised by Hintikka [7] and Barwise [1] who showed that branching indeed happens in natural language and how quantifier scope ambiguities are manifestations of it.

The set-theoretic semantics of branching quantifiers is due to Barwise [1] and it is as follows. Suppose $\llbracket v \rrbracket \subseteq \llbracket n_1 \rrbracket \times \llbracket n_2 \rrbracket$, then we have the following two cases:

$$\begin{cases} d_1, d_2 \text{ upward monotone} & \exists X \subseteq \llbracket n_1 \rrbracket, \exists Y \subseteq \llbracket n_2 \rrbracket, d_1 \llbracket n_1 \rrbracket X \ \& \ d_2 \llbracket n_2 \rrbracket Y \ \& \ X \times Y \subseteq \llbracket v \rrbracket \\ d_1, d_2 \text{ downward monotone} & \exists X \subseteq \llbracket n_1 \rrbracket, \exists Y \subseteq \llbracket n_2 \rrbracket, d_1 \llbracket n_1 \rrbracket X \ \& \ d_2 \llbracket n_2 \rrbracket Y \ \& \ X \times Y \supseteq \llbracket v \rrbracket \end{cases}$$

The general form of branching in our setting is the following left hand side diagram:



This diagram is very similar to the branching diagram. We have tried to make this apparent by rotating it at the right hand side above. This diagram on its own does not say much until we specify what is inside the $\llbracket v \rrbracket$ triangle. It is according to the content of $\llbracket v \rrbracket$ that the quantifiers interact with each other. We can encode Barwise's definition in the triangle when defining concrete interpretations for $\llbracket v \rrbracket$ in any of the instantiations. In the relational instantiation, $\llbracket v \rrbracket$ is an element of a rank 3 tensors of type $\mathcal{P}(U) \otimes \{\star\} \otimes \mathcal{P}(U)$, and $\llbracket n_1 \rrbracket$ and $\llbracket n_2 \rrbracket$ are elements of objects of type $\mathcal{P}(U)$. Thus in the above diagram, W will be instantiated as $\mathcal{P}(U)$ and S as $\{\star\}$. Given these, we define:

$$\begin{cases} d_1, d_2 \text{ upward monotone} & \star \llbracket v \rrbracket (X, \star, Y) \iff \llbracket v \rrbracket (X) \subseteq Y \\ d_1, d_2 \text{ downward monotone} & \star \llbracket v \rrbracket (X, \star, Y) \iff \llbracket v \rrbracket (X) \supseteq Y \end{cases}$$

where $\llbracket v \rrbracket (X)$ is the forward image of $\llbracket X \rrbracket$ in the binary relation $\llbracket v \rrbracket$. We do not need to explicitly talk about $X \subseteq \llbracket n_1 \rrbracket$ and $Y \subseteq \llbracket n_2 \rrbracket$, since the lines of the diagram are identities on the object $\mathcal{P}(U)$, hence they carry subsets. When a line emanates from the triangle that interprets $\llbracket n_1 \rrbracket$, it represents a subset of $\llbracket n_1 \rrbracket$, and when it emanates from the triangle that interprets $\llbracket n_2 \rrbracket$, it represents a subset of $\llbracket n_2 \rrbracket$. Generalizing this definition to the abstract categorical framework is work in progress.

7 Conclusion

In this paper, we reviewed our previous treatment of generalised quantifiers in the categorical compositional distributional semantics, where some objects have a bialgebra over them. In this paper, we showed how one can deal with quantifier scope ambiguity in that setting. Scope ambiguity gives rise to branching quantifiers. We showed how one may deal with branching in that setting as well. A compositional passage from syntax to scope ambiguity and to branching is however missing from the current treatment; it constitutes future work.

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