Entailment Relations on Distributions

J. van de Wetering

Department of Computer Science
University of Oxford

Workshop on Semantic Spaces at the Intersection of NLP, Physics and Cognitive Science, 2016

Lexical Entailment and Disambiguation

Entailment

"Dog bites cat"	$\Longrightarrow$	"Animal bites cat"
NOT "Animal bites cat"	$\Longrightarrow$	"Dog bites cat"

"Dog" *entails* "Animal"

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Disambiguation

bank	$\Longrightarrow$	river bank investment bank
cut	$\Longrightarrow$	cut of a movie a type of wound

Lexical Entailment and Disambiguation (2)

Properties:

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$\implies$ Try to capture this in a partial order on distributions

Information Ordering

Definition

An information ordering is a partial order on distributions with suitable information like properties

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Partial Order

A relation $\sqsubseteq$ such that for all x, y and z

- Reflexive: $x \sqsubseteq x$
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- Anti-symmetric: $x \sqsubseteq y$ and $y \sqsubseteq x$ implies $x = y$

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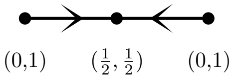
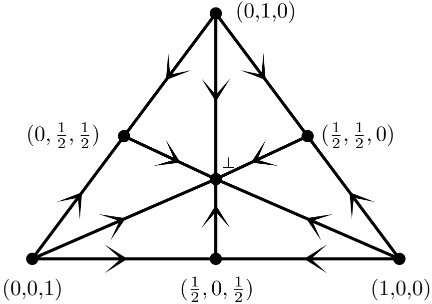
- Maximal elements: Pure states $\top_i = (0, \ldots, 0, 1, 0 \ldots, 0)$

Information flow in low dimensions



(a) Information flow for $n = 2$



(b) Information flow for $n = 3$

- Information order is unique for $n = 2$

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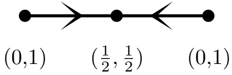

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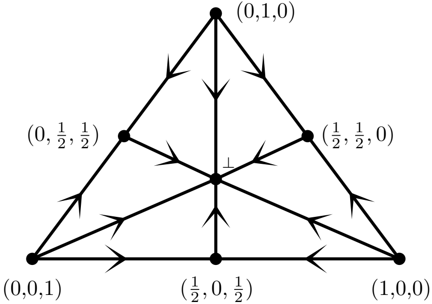
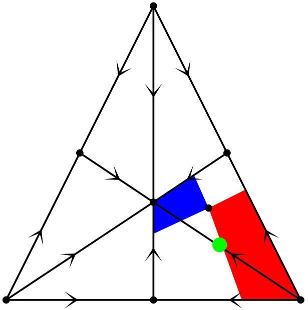


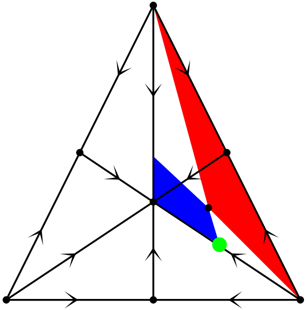
(b) Information flow for $n = 3$

- Information order is unique for $n = 2$
- Not unique for $n = 3$ and higher!

Renormalised Löwner orders

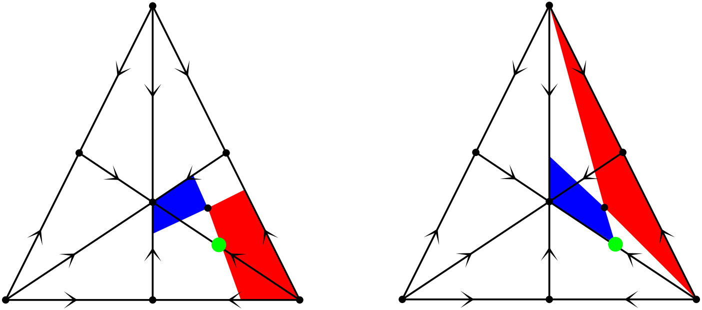


(a) $\sqsubseteq_L^+$: Renormalised to highest eigenvalue



(b) $\sqsubseteq_L^-$: Renormalised to lowest eigenvalue

Renormalised Löwner orders



(a) $\sqsubseteq_L^+$: Renormalised to highest eigenvalue

(b) $\sqsubseteq_L^-$: Renormalised to lowest eigenvalue

If $y = \frac{1}{30}(15, 10, 5)$ and $x = \frac{1}{10}(6, 2, 2)$
Then: $y \sqsubseteq_L^+ x$ and $x \sqsubseteq_L^- y$

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- An extra restriction: The *degeneracy* condition
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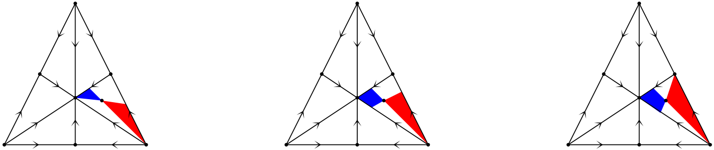

(a) The minimal order (b) Bayesian order (c) The maximal order

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- No contradiction with $\sqsubseteq_L^+$ and $\sqsubseteq_I^-$
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- Restricted information orders might prove too... restrictive
- There is a natural way to define grading
- Other techniques for smoothing/grading

Summary

- There is an analog between entailment/disambiguation and information content
- There is a large selection of possible partial orders that could encode this
- Empirical research needed
- Compositionality?